

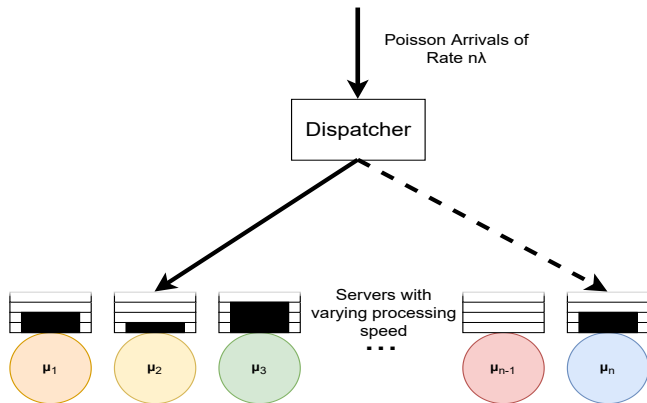
(Refined) Mean Field Approximation for Heterogeneous Interaction Models

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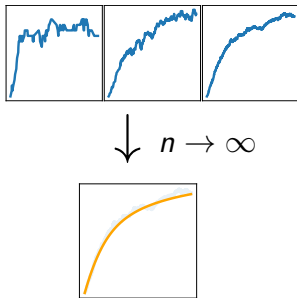
POLARIS

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Motivation



Classical Mean Field Setting



Mean Field methodology:

- $M_s^{(n)}(t) = \frac{1}{n} \{\# \text{ objects in state } s \text{ at time } t\}$
- $\lim_{n \rightarrow \infty} M_s^{(n)}(t) = \text{ODE}$

Works for: - objects w/ homogeneous transitions
- groups of objects w/ similar statistical behavior

Need to Model Heterogeneity

Importance of Heterogeneity

- heterogeneity has a dramatic impact
e.g. for caching, epidemic modelling, load balancing
- many homogeneous models ignore heterogeneity
- in general **no theoretical guarantees** for accuracy

Key Question

Can we adapt and justify the mean field approximation as valid technique?

Results

$X^{(n)}(t) \sim$ Stochastic System

$X_{(k,s)}^{(n)}(t) \sim$ indicates if item k is in state s as time t

$$\begin{aligned} \frac{d}{dt} \mathbb{E}[X_{(k,s)}^{(n)}(t)] &= \dots \mathbb{E}[X_{(k,s)}^{(n)}(t) X_{(k',s)}^{(n)}(t)] \dots \\ &\stackrel{\text{MF Assumption}}{\approx} \dots \mathbb{E}[X_{(k,s)}^{(n)}(t)] \mathbb{E}[X_{(k',s)}^{(n)}(t)] \dots \Rightarrow \frac{d}{dt} \phi_{(k,s)}(t) \end{aligned}$$

Theorem [A., Gast, ACM SIGMETRICS 22]*

For the Mean Field approximation $\phi(t)$ and refinement term $\mathbf{v}(t)$

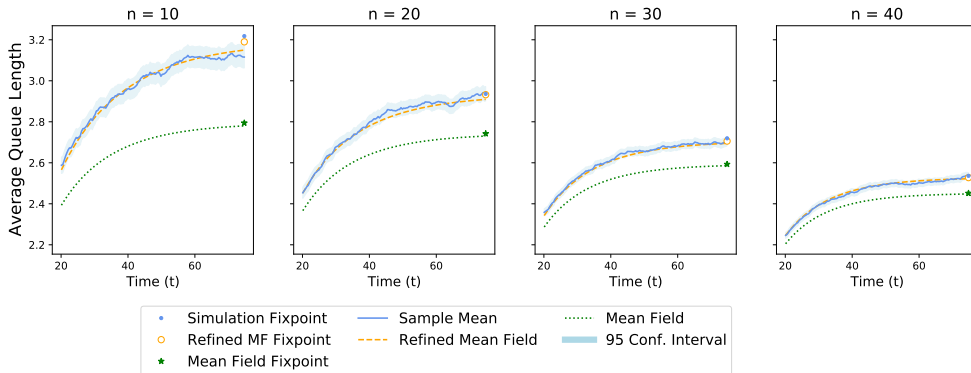
$$\mathbb{P}(\text{item } k \text{ in state } s \text{ at time } t) = \phi_{(k,s)}(t) + O(1/n),$$

$$\mathbb{P}(\text{item } k \text{ in state } s \text{ at time } t) = \phi_{(k,s)}(t) + v_{(k,s)}(t) + O(1/n^2).$$

* Mean Field and Refined Mean Field Approximations for Heterogeneous Systems: It Works! - Proc. ACM Meas. Anal. Comput. Syst., (Feb 2022)

Increasing Mean Field Accuracy as n grows

Almost exact Refined MF*



* implemented using `rmf_tool` – a library to compute (refined) mean field approximation(s) – SIGMETRICS Perform. Eval. Rev. (Workshop)